Course No. N 986, N986B, N987

Orange Unified School District

CALCULUS AB/BC AP

Year Course

GRADE LEVEL: 10-12

PREREQUISITES: Pre-Calculus with Trigonometry with grade of C or better

INTRODUCTION TO THE SUBJECT:

Advanced Placement Calculus is a year course equivalent to the first year of college calculus at most colleges and universities. The course is designed to prepare students to take the Mathematics Advanced Placement Examination, AB or BC level. Colleges and universities give advanced placement and/or college credit based on the results of the AP examination. Students are encouraged to take the Advanced Placement Examination. Areas of study include: functions, limits, continuity, derivative, integral, and series. The graphing calculator is used extensively in the course and on the AP examinations.

COURSE OBJECTIVES:

BY THE END OF THE COURSE THE STUDENT WILL BE ABLE TO:

Work with functions represented graphically, numerically, analytically, or verbally.

Understand the derivative in terms of a rate of change and local linear approximation.

Work with the derivative to solve a variety of application problems.

Understand the meaning of the definite integral both as a limit of Riemann sums and as the net accumulation of the rate of change.

Work with integrals to solve a variety of application problems.

Understand the relationship between the derivative and the definite integral as expressed in the Fundamental Theorem of Calculus.

Communicate solutions to mathematics problems in oral and written form.

Write a description of a physical situation with a function, a differential equation, or an integral.

Use a graphing calculator to help solve problems, experiment, interpret results, and verify conclusions.

Determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement.

Develop an appreciation of calculus as a body of knowledge and as a human accomplishment.

COURSE OVERVIEW AND APPROXIMATE UNIT TIME ALLOTMENTS:

FIRST SEMESTER — **WEEKS**

I. Functions, Graphs, and Limits: — 9
- A. Analysis of graphs
- B. Limits of functions (including one-sided limits)
  1. Calculating limits using algebra.
  2. Estimating limits from graphs or tables of data.
- C. Asymptotic and unbounded behavior
  1. Understanding asymptotes in terms of graphical behavior.
  2. Describing asymptotic behavior in terms of limits involving infinity.
  3. Comparing relative magnitudes of functions and their rates of change. (Example: contrasting exponential, polynomial, and logarithmic growth)
- D. Continuity as a property of functions
  1. Understanding continuity in terms of limits.
  2. Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem).
- E.* Parametric, polar, and vector functions

II. Derivatives: — 9
- A. Concepts of the derivative
  1. Derivative defined as the limit of the different quotient.
  2. Relationship between differentiability and continuity.
- B. Derivative at a point
  1. Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
  2. Tangent line to a curve at a point and local linear approximation.
  3. Instantaneous rate of change as the limit of average rate of change.
  4. Approximate rate of change from graphs and tables of values.

WEEKS

C. Derivative as a function
   1. Corresponding characteristics of graphs of f and f'.
   2. Relationships between the increasing and decreasing behavior of f and the sign of f'.
   3. The Mean Value Theorem and its geometric consequences.
   4. Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

D. Second derivatives
   1. Corresponding characteristics of the graphs of f, f', f''.
   2. Relationship between the concavity of f and the sign of f''.
   3. Point of inflection as place where concavity changes.

E. Applications of derivatives
   1. Analysis of curves, including the notions of monotonicity and concavity.
   2. Optimization, both absolute and relative extrema.
   3. Modeling rates of change, including related rates problems.
   4. Use of implicit differentiation to find the derivative of an inverse function.
   5. Interpretation of the derivative as a rate of change in varied applied contexts, including speed, velocity, and acceleration.
   6.* Analysis of planar curves given in parametric form, polar form, and vector form, including velocity and acceleration vectors.
   7.* Geometric interpretation of differential equations via slope fields and the relationship between slope fields and derivatives of implicitly defined functions.
   8.* Numerical solutions of differential equations using Euler’s method.
   9.* L’ Hopital’s Rule and its use in determining convergence of improper integrals and series.

F. Computation of derivatives
   1. Knowledge of derivatives of basic functions, including x^r, exponential, logarithmic, trigonometric, and inverse trigonometric functions.
   2. Basic rules for the derivative of sums, products, and quotients of functions.
   3. Chain rule and implicit differentiation.
   4.* Derivative of parametric, polar, and vector functions.

*Topics are only on the BC level exam

SECOND SEMESTER **WEEKS**

III. Integrals: 18

- A. Riemann sums
  1. Concept of a Riemann sum over equal subdivisions.
  2. Computation of Riemann sums using left, right, and midpoint evaluation points.
- B. Interpretations and properties of definite integrals
  1. Definite integral as a limit of Riemann sums.
  2. Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval: $\int_a^b f'(x)dx=f(b)-f(a)$.
  3. Basic properties of definite integrals. (For example, additivity and linearity.)
- C. Applications of integrals
  1. Area of a region under a curve.
  2. Volume of a solid with known cross sections.
  3. Average value of a function.
  4. Distance traveled by a particle along a line.
  - 5.* Length of a curve (including a curve given in parametric form).
  - 6.* Area of a region bounded by polar curve.
- D. Fundamental Theorem of Calculus
  1. Use of the Fundamental Theorem to evaluate definite integrals.
  2. Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined.
- E. Techniques of antidifferentiation
  1. Antiderivatives following directly from derivatives of basic functions.
  2. Antiderivatives by substitution of variables (including change of limits for definite integrals).
  - 3.* Antiderivative by parts and simple partial fractions (non repeating linear factors only).
  - 4.* Improper integrals (as limits of definite integrals).
- F. Applications of antidifferentiation
  1. Finding specific antiderivative using initial conditions, including applications to motion along a line.
  2. Solving separable differential equations and using them in modeling. In particular, studying the equation $y'=ky$ and exponential growth.

WEEKS

3.* Solving logistic differential equations and using them in modeling.

G. Numerical approximations to definite integrals

IV.* Polynomical approximations and series: (Calculus BC only)

A. Concept of series

B. Series of constants

1. Motivating examples including decimal expansion.
2. Geometric series with applications.
3. The harmonic series.
4. Alternating series with error bound.
5. Terms of series as areas of rectangles and their relationship to improper integrals, including the integral test and its use in testing the convergence of p-series.
6. The ration test for convergence or divergence.
7. Comparing series to test for convergence or divergence.

C. Taylor series

1. Taylor polynomial approximation with graphical demonstration of convergence. (For example, viewing graphs of various Taylor polynomials of the sine function approximating the sine curve.)
2. The general Taylor series centered at x=a.
3. Maclaurin series for the functions e^x, sinx, cosx, and $\frac{1}{1-x}$.
4. Formal manipulation of Taylor series and shortcuts to computing Taylor series, including differentiation, antidifferentiation, and the formation of new series from known series.
5. Function defined by power series and radius of convergence.
6. Lagrange error bound for Taylor polynomials.

*Topics are only on the BC level exam

DATE OF CONTENT REVISION: October 1997

DATE OF BOARD APPROVAL: November 24, 1997