A Look at...

Kindergarten
in California Public Schools

Including information about the new
Common Core State Standards

Overview

Effective mathematics education provides students with a balanced instructional program. In such a program, students become proficient in basic computational skills and procedures, develop conceptual understandings, and become adept at problem solving. Standards-based mathematics instruction starts with basic material and increases in scope and content as the years progress. It is like an inverted pyramid, with the entire weight of the developing subject, including readiness for algebra, resting on the foundations built in the early grades.

California recently adopted new standards in mathematics, the Common Core State Standards (CCSS) with California additions. The CCSS are comprised of standards developed by the state-led Common Core State Standards Initiative and material taken from the 1997 California mathematics standards. California will implement these new standards gradually over the next several years as curriculum frameworks, instructional materials, and assessments based on the CCSS are adopted.

There are many similarities between the CCSS and the 1997 California mathematics standards, but there are also a few noteworthy differences. For instance, the CCSS are organized by "domains" which add grade-level focus and vary slightly by grade. The domains for kindergarten are Counting and Cardinality (CC), Operations and Algebraic Thinking (OA), Number and Operations in Base Ten (NBT), Measurement and Data (MD), and Geometry (G). Also, the CCSS do not include "key standards" as in the 1997 California mathematics standards. Instead the CCSS are designed to have a greater focus at each grade and to develop mathematics topics in depth. In the early grades, the CCSS continue to emphasize concepts necessary for the study of more advanced mathematics in later years. To ensure that students have adequate time to achieve mastery, some of the 1997 California mathematics standards familiar to California's kindergarten teachers will be taught in different grades after the CCSS are fully implemented.

This section provides an overview of the new CCSS for kindergarten mathematics, including some highlights of how the kindergarten curriculum, based on the 1997 California mathematics standards, will change with the implementation of the new CCSS. It includes a review of some mathematical concepts for entering kindergarteners to know and guidance on areas of mathematics that may be challenging for some English learners. A complete listing of the kindergarten CCSS for mathematics can be found at the end of this section. A complete listing of the kindergarten 1997 California mathematics standards is located on the CDE Content Standards Web page at http://www.cde.ca.gov/be/st/ss/documents/mathstandard.pdf.

What Kindergarten Students Should Know

Kindergarten is a critical time for children. Students entering kindergarten can vary in age from four to six years old and bring varied life experiences, social skills, and physical and intellectual developmental characteristics. Participation in a quality preschool program is one of the best ways to prepare a child for success in kindergarten and beyond.

The *California Preschool Learning Foundations, Volume 1* (2008) describe the knowledge, skills, and competencies that children typically attain at around 48 and 60 months of age when they participate in a high-

quality preschool program and with adequate support. Such preschool programs promote student learning in mathematics by focusing on the mathematics in a child's everyday environment. For example, preschool children are introduced to concepts and relationships of numbers and quantities in their everyday environment as they recite the numbers in order to ten, count up to five objects, or visually compare two groups of objects and communicate if they are the "same" or "more." Children learn about measurement by comparing the length, weight, or capacity of objects by using words such as bigger, longer, heavier, or taller. Children learn these important foundations of mathematics while engaging in imaginative play, exploring the environment and materials, making discoveries, or interacting with teachers or other adults.

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Students are better prepared if they enter kindergarten with some background in the academic language of mathematics (the language of tests and texts) and an understanding of the concepts represented by such language. Students ready for school should have an understanding of mathematical attributes, such as color, shape, and size; abstract concepts, such as *some, all,* and *none;* and ordinal concepts, such as *before, after, yesterday,* and *tomorrow*. In addition, students who know the concepts in their native language but do not yet know the English words for the concepts will need extra support from teachers.

Fortunately, kindergarten provides many opportunities to support the development of critical mathematics vocabulary and concepts during both instructional time and playtime. For example, students learn to take turns during a game or line up for recess (first, second, third), count off in a line (one, two, three), or students learn to match the number of balls available for recess to the same number of children (matching sets).

What Students Learn in Kindergarten

In kindergarten, students are introduced to the relationship between numbers and quantities and build a foundation for place value as they count, represent and compare whole numbers, initially with sets of objects. Students also describe and model objects in their environment using simple geometric shapes and vocabulary.

Counting and Cardinality

Both the 1997 California mathematics standards and the CCSS for kindergarten focus on understanding the relationship between numbers and quantities. Kindergarteners learn the number names as they count (to 100 by 1s and 10s) and write number names (from 0 to 20). Students learn that each successive number name refers to a quantity that is one larger as they count objects and say the corresponding number names. Kindergarteners count objects (as many as 20) to answer "how many?" questions and they group and compare sets of concrete items (up to 10 objects in each group) to identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group.

At this age, some children may have difficulty with the coordination needed to write numerals (from 0 to 20) as called for in the standards. To help develop their writing skills, students may copy a numeral many times, then write the numeral with some prompts (e.g., dots or arrows), and finally write it from memory as the teacher says the number. A multisensory approach is important at this age and students may need to be encouraged to not be concerned about the quality of their handwriting as they learn to write numerals.

K.17

With full implementation of the CCSS, kindergarteners will extend counting to 100 by ones and tens (a grade one topic in the 1997 California mathematics standards).

Operations and Algebraic Thinking

Both the 1997 California mathematics standards and the CCSS introduce simple addition and subtraction in kindergarten. Kindergarteners use a variety of approaches (e.g., use of objects, fingers, drawings, sounds, verbal explanations or equations) to represent addition and subtraction (putting together and taking apart) and to solve problems (within 10). They decompose numbers (less than or equal to 10) into various pairs (e.g., $5 = 2 + 3$ and $5 = 4 + 1$) and find the missing number that makes 10 (for any number from 1 to 9). Kindergarteners will develop fluency with addition and subtraction (within 5).

Number and Operations in Base Ten

The CCSS introduce kindergarten students to the foundations for place value. Students use objects or drawings to compose and decompose numbers (from 11 to 19) into ten ones and some further ones (e.g., $18 = 10 + 8$). In the 1997 California mathematics standards, the concept of place value is covered in a similar way as kindergartens use estimation strategies in computation and problem solving for numbers in the ones and tens places.

Measurement and Data

Both the 1997 California mathematics standards and the CCSS provide opportunities for students to develop their measurement and classifying skills. Kindergarteners directly compare objects with measureable attributes (such as length or weight) to see which object is longer, shorter, lighter, heavier, or in general have "more of" / "less of" an attribute. Students also classify objects into categories and sort the categories by count.

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In both the 1997 California mathematics standards and the CCSS kindergarten students study the concepts of time and the tools that measure time (e.g., clock, calendar). Students will need repeated practice to memorize the sequence of the days of the week and months of the year. A firm understanding of these items of the calendar will help students avoid difficulty with other important concepts of time such as before and after.

With full implementation of the CCSS, skills associated with extending simple patterns and collecting and reporting data, part of the 1997 California mathematics standards at kindergarten, will be introduced at grade one.

Geometry

In both the 1997 California mathematics standards and the CCSS, students describe objects in the environment using the names of shapes (e.g., squares, circles, spheres) and identify shapes as two-dimensional (flat) and three-dimensional (solid).

Full implementation of the CCSS will further develop geometry skills as kindergarten students describe the relative positions of objects (e.g., above or behind), which is a topic in the 1997 California mathematics standards at grade one. The CCSS calls for kindergarteners to "model shapes in the world" by building and drawing shapes and also to compose simple shapes to form larger shapes (e.g., triangles to form rectangles). The concept of putting shapes together and taking them apart is a topic in the 1997 California mathematics standards at grade two.

Support for English Learners

Students need to develop knowledge of mathematics as a language. However, the academic language of mathematics instruction and the specialized vocabulary of mathematics can create particular challenges for English learners.

The language of mathematics is very precise compared with the English used in common discourse. English learners need opportunities to develop their knowledge of the features of language that are used to teach mathematics, such as semantics (how to translate the words of a problem into a symbolic representation), syntax (the order of words and phrases), and mathematical discourse (writing or talking about mathematical terms, concepts, etc.). The specialized vocabulary of mathematics should be explicitly taught and reinforced throughout the year.

These areas can create special challenges for English learners in the early grades:

- At an early stage students may have difficulty with such English words as first, second, last, before, every, each, more, and equal. Students may be unfamiliar with sum, difference, solve, length, and value.

- The different meanings of multiple-meaning words should be explicitly taught. These words may have a meaning in common discourse that is different from the meaning in mathematics, such as table or face (as in the face of a clock).

- The place values of some of the numbers between 10 and 20 are not obvious from their names (e.g., the number 16 is called sixteen in English, but ten plus six in other languages).

- The narrative descriptions of a word problem can require language skills that students have not yet mastered, particularly when the language of a word problem is ambiguous or includes idioms (e.g., "a dime a dozen"), comparatives (greater than, less than, most often, least often), or position words (behind, below, in front of, to the right or left of).

Instruction in mathematics should be promoted despite low literacy or limited proficiency in the English language, along with critical thinking and analysis skills. Specially designed academic instruction in English (SDAIE) strategies can provide valuable instructional strategies to meet the needs of English learners. For additional resources to support the teaching of English learners, go to the CDE English Learners Web page at http://www.cde.ca.gov/sp/el/.

Transition to Common Core State Standards

The following chart highlights a few topics that will continue to be addressed at the grade level and some of the changes to be considered as California progresses toward full implementation of the kindergarten CCSS for mathematics. The chart includes the column heading "Overview of Standards." For the 1997 California mathematics standards, this information is from the "strands" (e.g., Number Sense) and the "overarching" standards (e.g., Number Sense 1.0) at kindergarten. For the CCSS, the column lists the "domains" (e.g., Operations and Algebraic Thinking) and the "cluster headings" for the standards (e.g., Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from) at kindergarten.

The chart does not, and is not intended to, illustrate all of the differences between the two sets of standards—it is merely a beginning point for more in-depth discussion by teachers and other educators on how instruction may change.

The transition chart is followed by a complete set of the CCSS with California additions for kindergarten and then a table of the CCSS domains for kindergarten through grade five.

A Quick Look: Transition to Common Core State Standards (CCSS)

Mathematics: Kindergarten

Overview of Standards 1997 California Mathematics Standards*	Overview of Standards CCSS	Highlights
Algebra and Functions ■ Students sort and classify objects. **Number Sense** ■ Students understand the relationship between numbers and quantities (i.e., that a set of objects has the same number of objects in different situations regardless of its position or arrangement). ■ Students understand and describe simple additions and subtractions. ■ Students use estimation strategies in computation and problem solving that involve numbers that use the ones and tens places.	**Counting and Cardinality** ■ Know number names and the count sequence. ■ Count to tell the number of objects. ■ Compare numbers. **Operations and Algebraic Thinking** ■ Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.	■ Introduce counting to 100 by 1s and 10s **(counting from 30 to 100 and introduction to skip counting by 10s moves from grade one to kindergarten in CCSS). ▼**[**] ■ Represent a number of objects with a written numeral 0–20. ■ Count objects to understand the relationship between numbers and quantities and to answer "how many" questions for numbers from 1–20. ■ Identify if the number of objects in one group is greater than, less than or equal to the number of objects in another (for groups with up to 10 objects). ■ Focus on representing addition and subtraction in various ways such as using objects, fingers, drawings, verbal explanations, or equations ■ Add and subtract and solve addition and subtraction word problems for numbers within 10, by using objects or drawing. ■ Fluently add and subtract within 5. ■ Decompose numbers (less than or equal to 10) into pairs.

Measurement and Geometry

- Students understand the concept of time and units to measure it; they understand that objects have properties, such as length, weight, and capacity, and that comparisons may be made by referring to those properties.
- Students identify common objects in their environment and describe the geometric features.

Statistics, Data Analysis, and Probability

- Students collect information about objects and events in their environment.

Number and Operations in Base Ten

- Work with numbers 11–19 to gain foundations for place value.

Measurement and Data

- Describe and compare measurable attributes.
- Classify objects and count the number of objects in categories.

Geometry

- Identify and describe shapes.
- Analyze, compare, create, and compose shapes.

- Introduce composing and decomposing numbers from 11–19 into tens and ones.

- Directly compare two objects and describe the differences based on a measureable attribute in common (e.g., height).
- Classify objects into given categories and sort by count.
- Understand concepts of time (e.g., morning, today, week) and tools that measure time (e.g., clock, calendar). Name the days of the week and identify time of everyday events (e.g., lunch time is 12 o'clock).
- Introduce collecting data and recording results **(moves from kindergarten to grade one in the CCSS).** ▲
- Introduce simple patterns **(moves from kindergarten to grade one in the CCSS).** ▲

- Correctly name and identify shapes as two-dimensional (flat) and three-dimensional (solid).
- Analyze and compare two- and three-dimensional shapes, using informal language.
- Model shapes by building and drawing shapes **(a new focus in the CCSS).**
- Compose simple shapes to form larger shapes, such as triangles to form rectangles **(putting shapes together moves from grade two to kindergarten in the CCSS).** ▼
- Describe the relative positions of objects, such as above or behind **(moves from grade one to**

- The CCSS include Standards for Mathematical Content (different at each grade) and Standards for Mathematical Practice (recurring throughout the grades).

- To master the grade level content, students will need to rely on their understanding of a concept and not only on procedures. Standards for Mathematical Practice define how students develop mathematical understanding as they make sense of a problem, reason abstractly, construct arguments, model with mathematics, use tools strategically, attend to precision, and look for structure and repeated reasoning.

- Standards for Mathematical Content that set an expectation of "understanding" are potential points of intersections between these standards and the Standards for Mathematical Practice.

- Standards for Mathematical Practice are similar to the previous 1997 California Mathematical Reasoning standards and should be evident throughout future curricula, assessments and professional development.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning

Mathematical Reasoning

- Students make decisions about how to set up a problem.

- Students solve problems in reasonable ways and justify their reasoning.

The Standards

The CCSS with California additions that follow are the pre-publication version of the standards prepared by the Sacramento County Office of Education (SCOE), updated on October 21, 2010. Content that is unique to California and was added to the multi-state common core standards is in bold typeface. The SCOE document is available online at http://www.scoe.net/castandards/agenda/2010/math_ccs_recommendations.pdf. These kindergarten CCSS for mathematics were adopted by the California State Board of Education on August 2, 2010.

A complete listing of the kindergarten 1997 California mathematics standards is located on the CDE Content Standards Web page at http://www.cde.ca.gov/be/st/ss/documents/mathstandard.pdf.

Common Core State Standards with California Additions Mathematics – Kindergarten	
Counting and Cardinality (K.CC)	
Know number names and the count sequence.	
1.	Count to 100 by ones and by tens.
2.	Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
3.	Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).
Count to tell the number of objects.	
4.	Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger.
5.	Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.
Compare numbers.	
6.	Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[1]
7.	Compare two numbers between 1 and 10 presented as written numerals.

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[1] Include groups with up to ten objects.

	Operations and Algebraic Thinking (K.OA)
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.
1.	Represent addition and subtraction with objects, fingers, mental images, drawings[2], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
2.	Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
3.	Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $5 = 2 + 3$ and $5 = 4 + 1$).
4.	For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
5.	Fluently add and subtract within 5.
Number and Operations in Base Ten (K.NBT)
Work with numbers 11–19 to gain foundations for place value.
1.	Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., $18 = 10 + 8$); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.
Measurement and Data (K.MD)
Describe and compare measurable attributes.
1.	Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.
2.	Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*
Classify objects and count the number of objects in each category.
3.	Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.[3]
4.	**Demonstrate an understanding of concepts time (e.g., morning, afternoon, evening, today, yesterday, tomorrow, week, year) and tools that measure time (e.g., clock, calendar). (CA-Standard MG 1.2)** **a. Name the days of the week. (CA-Standard MG 1.3)** **b. Identify the time (to the nearest hour) of everyday events (e.g., lunch time is 12 o'clock, bedtime is 8 o'clock at night). (CA-Standard MG 1.4)**

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[2] Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

[3] Limit category counts to be less than or equal to 10.

Geometry (K.G)
Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

1.	Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above, below, beside, in front of, behind,* and *next to*.
2.	Correctly name shapes regardless of their orientations or overall size.
3.	Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").

Analyze, compare, create, and compose shapes.	
4.	Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).
5.	Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.
6.	Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*
	Standards for Mathematical Practice Integrated throughout the CCSS 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

CCSS Domains

The CCSS are organized by domains. The table lists the domains for kindergarten through grade eight. The table identifies which domains are addressed in kindergarten through grade five (an "X" indicates the domain addressed at a grade level). The shaded rows indicate domains to be covered at later grades.

Domains	Kindergarten	Grade One	Grade Two	Grade Three	Grade Four	Grade Five
Counting and Cardinality (CC)	X					
Operations and Algebraic Thinking (OA)	X	X	X	X	X	X
Number and Operations in Base Ten (NBT)	X	X	X	X	X	X
Measurement and Data (MD)	X	X	X	X	X	X
Geometry (G)	X	X	X	X	X	X
Number and Operations – Fractions (NF)				X	X	X
Ratios and Proportional Relationships (RP)						
The Number System (NS)						
Expressions and Equations (EE)						
Statistics and Probability (SP)						
Functions (F)						